



HEAT TRANSFER ANALYSIS OF CONVECTIVE ANNULAR FINS WITH A STEP CHANGE IN THICKNESS AND VARIABLE THERMAL CONDUCTIVITY

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Abstract: The Adomian decomposition method has been used to determine the temperature distribution within annular fins with temperature-dependent thermal conductivity and a step reduction in thickness towards the fin tip. The method which provides an analytical solution in the form of an infinite power series is useful and practical for solving the resulting two nonlinear heat diffusion equations with nonlinear inhomogeneous boundary conditions. The fin temperature distribution has been obtained as a function of Biot number, geometrical fin parameters, thermal conductivity parameter describing the variation of the thermal conductivity, and thickness parameter which describes the variation of fin thickness. The results show that the temperature profile has an abrupt change in the temperature gradient where the step change in thickness occurs and thermal conductivity parameter has an important role on the temperature profile and the heat transfer rate.

Keywords: Adomian decomposition method, Annular fin, Variable conductivity

DEĞİŞKEN ISI İLETİM KATSAYILI KADEMELİ DAİRESEL KANATLARIN ISIL ANALİZİ

Özet: Bu çalışmada, ısı iletim katsayısı değişken olan kademeli dairesel bir kanat içerisindeki sıcaklık dağılımını bulmak için Adomian ayrıştırma yöntemi kullanılmıştır. Sonsuz kuvvet serisi şeklinde bir analitik çözüm sağlayan Adomian ayrıştırma yöntemi, ısı iletim katsayısının değişken olması yüzünden doğrusal olmayan ısı yayılım denklemini çözmek için kullanışlı ve pratik bir yöntemdir. Kanat içindeki sıcaklık dağılımı; Biot sayısı, geometrik kanat parametreleri ve ısı iletim katsayısının değişimini tanımlayan ısı iletim parametresi ve kanat kalınlığındaki değişimi gösteren kalınlık parametresinin fonksiyonu olarak elde edilmiştir. Bu çalışmada elde edilen sonuçlar kanat kalınlığının değiştiği noktada sıcaklık gradyanında ani bir değişim olduğunu ve ısı iletim parametresinin sıcaklık profili ve ısı geçişi üzerinde önemli bir rol oynadığını göstermiştir.

Anahtar Kelimeler: Adomian yöntemi, Dairesel kanat, Değişken ısı iletim katsayısı

NOMENCLATURE

A_m	Adomian polynomials
h	heat transfer coefficient, $[W/(m^2K)]$
k	thermal conductivity, $[W/(mK)]$
k_a	thermal conductivity at ambient fluid temperature, $[W/(mK)]$
L	the highest order derivative
L^{-1}	inverse operator of L
M	convective fin parameter
N	nonlinear operator
r	radial coordinate, (m)
r_m	intermediate radius, (m)
r_i	inner radius of annular fin, (m)
r_o	outer radius of annular fin, (m)
R	remainder of the linear operator or dimensionless radial coordinate
R_{in}	the ratio of intermediate radius to inner radius
R_o	the ratio of outer radius to inner radius
t	unreduced half-thickness of fin, (m)
T	temperature, (K)
U	dimensionless fin volume, $(V/2\pi t^3)$

V	fin volume, (m^3)
δ	dimensionless unreduced fin thickness
ξ	dimensionless radial coordinate from the fin base
η	dimensionless radial coordinate from the reduction radius towards the fin tip
γ	the slope of the thermal conductivity-temperature divided by the intercept k_a
θ	dimensionless temperature
ϕ	dimensionless temperature
λ	thickness ratio which describes the variation of fin thickness
κ	a parameter describing variation of thermal conductivity

INTRODUCTION

Annular fins find numerous applications in compact heat exchangers, in specialized installations of single and double-pipe heat exchangers, in electrical apparatus in which generated heat must be efficiently dissipated,

on cylinders of air cooled internal-combustion engines, etc. The analysis of annular fins with common profile shapes namely, rectangular, triangular, concave and convex parabolic and hyperbolic are available in the literature (Kraus et al., 2001). Several papers (Yeh and Liaw, 1993; Razani and Zohoor, 1991; Sohrabpour and Razani, 1993) are devoted to finding the shape of fins that would minimize the volume for a given amount of heat dissipation. However, all of these studies consider negligible heat transfer from the tip, because their fins have sharp tips. Apparently, the sharp-tip fin design has the disadvantage that the resulting profiles are too difficult to manufacture. Several other studies (Son and Bar-Cohen, 1981; Li, 1983; Yeh, 1995) have been conducted to find the dimensions of a constant thickness straight fin that would maximize the heat dissipation for a given volume. Hollands and Stedman (1992) proposed a design of absorber plate fin with a step reduction in thickness towards the fin tip for saving in fin material. The results from this study indicated that roughly a 20 % reduction in fin material is possible. Aziz (1994) investigated the optimum dimensions of convecting rectangular fins with a step change in cross-sectional area. A similar profile has also been adopted for radial fins by Kundu and Das (2001). These studies, however, invoked the assumption of constant thermal conductivity. When fins dissipate heat by convection, the performance is significantly affected by variable thermal conductivity.

In the present work the concentric annular fins with a step change in thickness and temperature-dependent thermal conductivity are considered. Resulting two nonlinear heat transfer equations with nonlinear boundary conditions are solved by Adomian decomposition method which is used for solving various nonlinear fin problems (Chiu and Chen, 2002a; Chiu and Chen 2002b; Chiu and Chen 2003; Arslanturk, 2005a; Arslanturk 2005b). Linearization, approximation and assumption are unnecessary during the analytical processes of the decomposition method. The decomposition method offers many advantages over other methods, including faster convergence and higher accuracy, and it may be used to solve the problems associated with the complex conditions presented by fin boundaries and geometries (Chiu and Chen, 2002a). Using the temperature distribution, heat transfer rate from the fin is expressed as a function of convective fin parameters, M_1 and M_2 ; thermal conductivity parameter, κ ; the ratio of intermediate radius to inner radius, R_{in} ; thickness ratio describing variation of thermal conductivity, λ ; and Biot number, Bi . The results show that the thermal conductivity parameter has an important role on the temperature profile and the heat transfer rate.

PROBLEM DESCRIPTION

An annular fin with temperature-dependent thermal conductivity and a step change in thickness as shown in Figure 1 is considered in analyzing the fin parameters.

The fin is exposed to a convective environment at the constant ambient temperature T_a and heat transfer coefficient h . The base temperature T_b of the fin is constant. Since the fin is assumed to be thin, the temperature distribution within the fin does not depend on axial direction. The energy balance equations are given

$$2t \frac{d}{dr} \left[k(T_1) r \frac{dT_1}{dr} \right] - 2hr(T_1 - T_a) = 0, r_i < r < r_{in} \quad (1a)$$

$$2\lambda t \frac{d}{dr} \left[k(T_2) r \frac{dT_2}{dr} \right] - 2hr(T_2 - T_a) = 0, r_{in} < r < r_o \quad (1b)$$

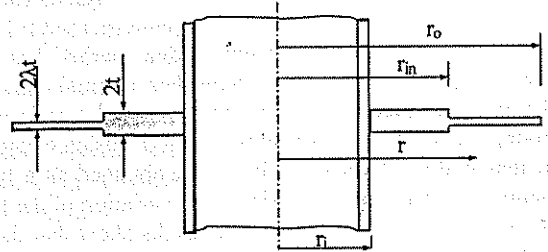


Figure 1. Schematic diagram of an annular fin with a step change in thickness

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$k(T) = k_a [1 + \gamma(T - T_a)] \quad (2)$$

where k_a is the thermal conductivity at the ambient fluid temperature of the fin, γ is the parameter describing the variation of thermal conductivity.

Employing the following dimensionless parameters

$$\begin{aligned} \xi &= \frac{r - r_i}{r_i} & \eta &= \frac{r - r_{in}}{r_{in}} & R &= \frac{r}{r_i} & R_{in} &= \frac{r_{in}}{r_i} & R_o &= \frac{r_o}{r_i} \\ \delta &= \frac{t}{r_i} & Bi &= \frac{hr_i}{k_a} & M_1 &= \sqrt{\frac{Bi}{\delta}} & M_2 &= \frac{M_1 R_{in}}{\sqrt{\lambda}} \\ \phi(\eta) &= \frac{T_2 - T_a}{T_b - T_a} & \theta(\xi) &= \frac{T_1 - T_a}{T_b - T_a} & \kappa &= \gamma(T_b - T_a) \\ U &= \delta[(1 - \lambda)R_{in} + \lambda R_o - 1] \end{aligned} \quad (3)$$

the formulation of the problem reduces to

$$\frac{d^2 \theta}{d\xi^2} = M_1^2 \theta - \kappa \theta \frac{d^2 \theta}{d\xi^2} - \kappa \left(\frac{d\theta}{d\xi} \right)^2 - \frac{\kappa}{(1 + \xi)} \theta \frac{d\theta}{d\xi} - \frac{1}{(1 + \xi)} \frac{d\theta}{d\xi}, \quad 0 < \xi < R_{in} - 1 \quad (4a)$$

$$\frac{d^2 \phi}{d\eta^2} = M_2^2 \phi - \kappa \phi \frac{d^2 \phi}{d\eta^2} - \kappa \left(\frac{d\phi}{d\eta} \right)^2 - \frac{\kappa}{(1 + \eta)} \phi \frac{d\phi}{d\eta} - \frac{1}{(1 + \eta)} \frac{d\phi}{d\eta}, \quad 0 < \eta < \frac{R_o}{R_{in}} - 1 \quad (4b)$$

$$\theta(0) = 1 \quad (4c)$$

$$\theta(R_{in} - 1) = \phi(0) \quad (4d)$$

$$\left[(1 + \kappa\theta) \frac{d\theta}{d\xi} + Bi(1 - \lambda)\theta \right]_{\xi=R_{in}-1} = \frac{\lambda}{R_{in}} \left[(1 + \kappa\phi) \frac{d\phi}{d\eta} \right]_{\eta=0} \quad (4e)$$

$$\left[(1 + \kappa\phi) \frac{d\phi}{d\eta} + R_{in} Bi\phi \right]_{\eta=\frac{R_o}{R_{in}}-1} = 0 \quad (4f)$$

THE ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition method, proposed by Adomian initially with the aims to solve frontier physical problem, has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, biology and chemical reactions etc. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly (Adomian, 1988; Adomian, 1994).

We consider a general nonlinear equation

$$Lu + Ru + Nu = g, \quad (5)$$

where L is the highest order derivative which is assumed to be easily invertible, R the linear differential operator of less order than L , Nu represents the nonlinear terms, and g is the source term. Applying the inverse operator L^{-1} to the both sides of Equation (5), and using the given conditions we obtain

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \quad (6)$$

where the function $f(x)$ represents the terms arising from integrating the source term $g(x)$, and from the using given conditions, all of which are assumed to be prescribed.

For nonlinear differential equations, the nonlinear operator $Nu = F(u)$ is represented by an infinite series of the so-called Adomian polynomials

$$F(u) = \sum_{m=0}^{\infty} A_m \quad (7)$$

The polynomials A_m are generated for all kind of nonlinearity so that A_0 depends only on u_0 , A_1 depends on u_0 and u_1 , and so on. The standard Adomian method defines the solution $u(x)$ by the series

$$u = \sum_{m=0}^{\infty} u_m \quad (8)$$

$F(u)$ is expanded Taylor series about u_0

$$F(u) = F(u_0) + F'(u_0)(u - u_0) + \frac{F''(u_0)}{2!} \frac{(u - u_0)^2}{2!} + \frac{F'''(u_0)}{3!} \frac{(u - u_0)^3}{3!} + \dots \quad (9)$$

and is rewritten Equation (8) as $u - u_0 = u_1 + u_2 + u_3 + \dots$, substituting it into Equation (9) and then equating two expressions for $F(u)$ found in Equation (9) and Equation (7) defines formulas for the Adomian polynomials

$$F(u) = A_1 + A_2 + \dots = F(u_0) + F'(u_0)(u_1 + u_2 + \dots) + F''(u_0) \frac{(u_1 + u_2 + \dots)^2}{2!} + \dots \quad (10)$$

By equating terms in Equation (10) the first few Adomian's polynomials $A_0 + A_1 + A_2 \dots$ are given

$$A_0 = F(u_0), \quad (11a)$$

$$A_1 = u_1 F'(u_0), \quad (11b)$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!} u_1^2 F''(u_0), \quad (11c)$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!} u_1^3 F'''(u_0), \quad (11d)$$

Now that the A_k are known, Equation (7) can be substituted in Equation (6) to specify the terms in the expansion for the solution of Equation (8).

FIN TEMPERATURE DISTRIBUTION

Following Adomian decomposition analysis, the linear operator is defined as:

$$L = \frac{d^2}{d\xi^2}. \quad (12)$$

Consequently, Equation (4a) can be written as follows:

$$L\theta = M_1^2\theta - \frac{1}{(1+\xi)} \frac{d\theta}{d\xi} - \kappa NA - \kappa NB - \frac{\kappa}{(1+\xi)} NC \quad (13)$$

where

$$NA = \theta \frac{d^2\theta}{d\xi^2} = \sum_{m=0}^{\infty} A_m \quad (14a)$$

$$NB = \left(\frac{d\theta}{d\xi} \right)^2 = \sum_{m=0}^{\infty} B_m \quad (14b)$$

$$NC = \theta \frac{d\theta}{d\xi} = \sum_{m=0}^{\infty} C_m \quad (14c)$$

are nonlinear terms. Hence, using Equation (11) results

$$A_0 = \theta_0 \frac{d^2\theta_0}{d\xi^2} \quad (15a)$$

$$A_1 = \theta_1 \frac{d^2\theta_0}{d\xi^2} + \theta_0 \frac{d^2\theta_1}{d\xi^2} \quad (15b)$$

$$A_2 = \theta_2 \frac{d^2\theta_0}{d\xi^2} + \theta_1 \frac{d^2\theta_1}{d\xi^2} + \theta_0 \frac{d^2\theta_2}{d\xi^2} \quad (15c)$$

$$A_3 = \theta_3 \frac{d^2\theta_0}{d\xi^2} + \theta_2 \frac{d^2\theta_1}{d\xi^2} + \theta_1 \frac{d^2\theta_2}{d\xi^2} + \theta_0 \frac{d^2\theta_3}{d\xi^2} \quad (15d)$$

$$B_0 = \left(\frac{d\theta_0}{d\xi} \right)^2 \quad (16a)$$

$$B_1 = 2 \frac{d\theta_0}{d\xi} \frac{d\theta_1}{d\xi} \quad (16b)$$

$$B_2 = \left(\frac{d\theta_1}{d\xi} \right)^2 + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_2}{d\xi} \quad (16c)$$

$$B_3 = 2 \frac{d\theta_1}{d\xi} \frac{d\theta_2}{d\xi} + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_3}{d\xi} \quad (16d)$$

and

$$C_0 = \theta_0 \frac{d\theta_0}{d\xi} \quad (17a)$$

$$C_1 = \theta_1 \frac{d\theta_0}{d\xi} + \theta_0 \frac{d\theta_1}{d\xi} \quad (17b)$$

$$C_2 = \theta_2 \frac{d\theta_0}{d\xi} + \theta_1 \frac{d\theta_1}{d\xi} + \theta_0 \frac{d\theta_2}{d\xi} \quad (17c)$$

$$C_3 = \theta_3 \frac{d\theta_0}{d\xi} + \theta_2 \frac{d\theta_1}{d\xi} + \theta_1 \frac{d\theta_2}{d\xi} + \theta_0 \frac{d\theta_3}{d\xi} \quad (17d)$$

:

Applying the inverse operator L^{-1} to both sides of Equation (13) one obtains

$$L^{-1}L\theta = M_1^2 L^{-1}\theta - L^{-1} \left[\frac{1}{(1+\xi)} \frac{d\theta}{d\xi} \right] - \kappa L^{-1}NA - \kappa L^{-1}NB - \kappa L^{-1} \left[\frac{1}{(1+\xi)} NC \right] \quad (18)$$

and

$$\theta = \theta_0 + M_1^2 L^{-1}\theta - L^{-1} \left[\frac{1}{(1+\xi)} \frac{d\theta}{d\xi} \right] - \kappa L^{-1}NA - \kappa L^{-1}NB - \kappa L^{-1} \left[\frac{1}{(1+\xi)} NC \right] \quad (19)$$

The inverse operator L^{-1} is twofold considered and 2-fold integral operator defined by

$$L^{-1} = \int_0^\xi \int_0^\xi (.) d\xi d\xi \quad (20)$$

$$L^{-1}L\theta = \theta - \theta(0) - \xi \frac{d\theta(0)}{d\xi} \quad (21)$$

thus,

$$\theta_0 = \theta(0) + \xi \frac{d\theta(0)}{d\xi} \quad (22)$$

The decomposition technique represents the solution of Eq. (13) as a series.

$$\sum_{m=0}^{\infty} \theta_m = \theta_0 + M_1^2 L^{-1} \left(\sum_{m=0}^{\infty} \theta_m \right) - L^{-1} \left[\frac{1}{(1+\xi)} \frac{d}{d\xi} \left(\sum_{m=0}^{\infty} \theta_m \right) \right] - \kappa L^{-1} \left(\sum_{m=0}^{\infty} A_m \right) - \kappa L^{-1} \left(\sum_{m=0}^{\infty} B_m \right) - \kappa L^{-1} \left[\frac{1}{(1+\xi)} \left(\sum_{m=0}^{\infty} C_m \right) \right] \quad (23)$$

Let $\frac{d\theta(0)}{d\xi} = c_1$, together with $\theta(0) = 1$,

$$\theta_0 = 1 + c_1 \xi \quad (24)$$

Consequently it can be written following recursive relationship, such as

$$\theta_{m+1} = M_1^2 L^{-1} \theta_m - L^{-1} \left[\frac{1}{(1+\xi)} \frac{d\theta_m}{d\xi} \right] - \kappa L^{-1} A_m - \kappa L^{-1} B_m - \kappa L^{-1} \left[\frac{1}{(1+\xi)} C_m \right] \quad m \geq 0 \quad (25)$$

Letting $\phi(0) = c_2$, $\frac{d\phi(0)}{d\eta} = c_3$ and applying the same procedure to Equation (4b), it can be written as follows:

$$\phi_0 = c_2 + c_3 \xi \quad (26)$$

$$\phi_{m+1} = M_2^2 L^{-1} \phi_m - L^{-1} \left[\frac{1}{(1+\xi)} \frac{d\phi_m}{d\xi} \right] - \kappa L^{-1} A_m - \kappa L^{-1} B_m - \kappa L^{-1} \left[\frac{1}{(1+\xi)} C_m \right] \quad m \geq 0 \quad (27)$$

HEAT TRANSFER RATE

Total heat transfer rate is expressed by applying the Fourier's law at the fin base, as

$$Q = -4\pi k_b r_1 t \left. \frac{dT_1}{dr} \right|_{r=r_1} \quad (28)$$

The dimensionless heat transfer rate is defined as follows:

$$q = \frac{Q}{4\pi k_a r_1 (T_b - T_a)} = -(1 + \kappa) \delta \left. \frac{d\theta}{d\xi} \right|_{\xi=0} \quad (29)$$

Since the temperature gradient at the fin base equal to the integral constant c_1 , the dimensionless heat transfer rate rewritten as

$$q = -(1 + \kappa) \delta c_1 \quad (30)$$

RESULTS AND DISCUSSION

The Adomian decomposition method provides an analytical solution in terms of an infinite series. The investigation on the convergence of this series can be seen in the literature for more general nonlinear problems (Cherruault, 1992; Cherruault et al, 1993). In order to make a comparison with the known results and to provide a useful test of the accuracy of the present method, the results are compared with the exact solution given by Kundu and Das (2001) for the case of constant thermal conductivity (see Table 1). Good agreement between the results was observed confirming the validity of the Adomian decomposition method.

Table 1. Comparison of Adomian decomposition method and analytical solution for the case of constant thermal conductivity ($R_{in}=1.5, R_o=2.0$)

Bi=0.01					Bi=0.10			
$\lambda=0.5, \delta=0.0047$			$\lambda=1.0, \delta=0.0033$		$\lambda=0.5, \delta=0.047$		$\lambda=1.0, \delta=0.033$	
R	Exact*	ADM	Exact*	ADM	Exact*	ADM	Exact*	ADM
1.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1.1	0.854150	0.854158	0.819271	0.819275	0.851624	0.851632	0.817925	0.817930
1.2	0.738402	0.738417	0.677861	0.677869	0.733519	0.733534	0.675249	0.675257
1.3	0.647023	0.647044	0.567358	0.567370	0.639872	0.639893	0.563505	0.563517
1.4	0.575688	0.575713	0.481488	0.481500	0.566291	0.566315	0.476374	0.476386
1.5	0.521115	0.521139	0.415531	0.415542	0.509432	0.509456	0.409096	0.409107
1.6	0.440696	0.440716	0.365931	0.365941	0.428699	0.428719	0.358072	0.358081
1.7	0.383386	0.383403	0.330014	0.330023	0.370598	0.370615	0.320588	0.320597
1.8	0.345241	0.345257	0.305795	0.305803	0.331178	0.331192	0.294616	0.294624
1.9	0.323490	0.323504	0.291836	0.291844	0.307636	0.307649	0.278672	0.278679
2.0	0.316301	0.316314	0.287148	0.287155	0.298089	0.298102	0.271714	0.271721

*Given by Kundu and Das (2001)

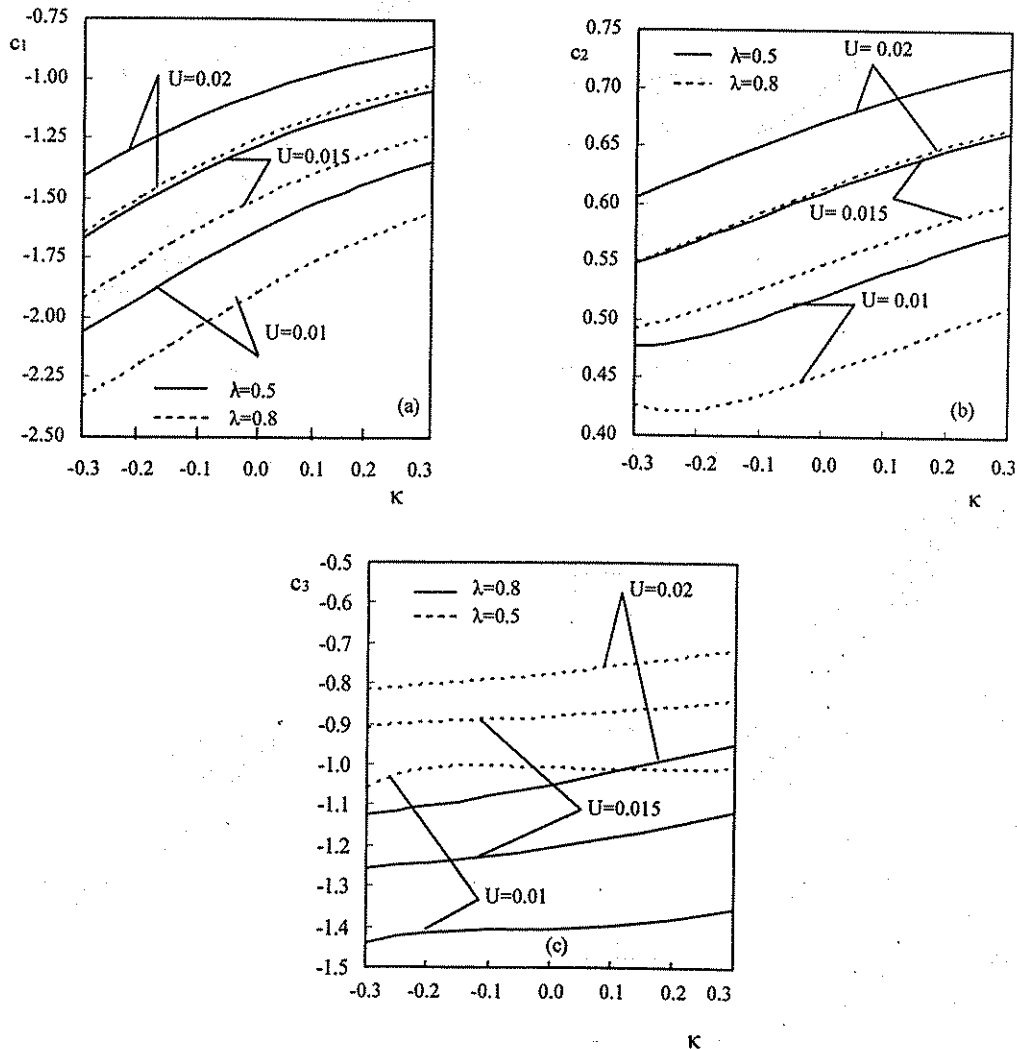


Figure 2. The integration constants versus thermal conductivity parameter for $Bi=0.01, R_{in}=1.5, R_o=2.0$, and selected values of fin volume U and thickness parameter λ

Integration constants c_1 , c_2 , and c_3 respectively represent the temperature gradient at the fin tip, the temperature and temperature gradient at the cross-section where the step change in thickness occurs. The constants can be evaluated from the boundary conditions given in Equations (4e)-(4f) using classical Newton-Raphson method. The values of the coefficients c_1 , c_2 , and c_3 relative to the thermal conductivity parameters, κ , thickness parameters, λ , and dimensionless fin volumes, U , are calculated and indicated in Figure (2). Figure (2a) shows that the values of the coefficient c_1 , i.e. temperature gradient at the fin base, increases with increasing thickness

parameter and decreasing thermal conductivity parameter while the fin volume is kept at a constant value. Figure (2b) presents the variation of c_2 , i.e. the temperature at the intermediate radius, with thermal conductivity parameter for different fin volume and thickness parameter. It can be stated that the intermediate temperature increases with increasing thermal conductivity and thickness parameters. From the Figure (2c), it can be seen that the coefficient c_3 , i.e. the absolute values of the temperature gradient at the intermediate radius, increases as the thickness parameter increases and thermal conductivity parameter decreases.

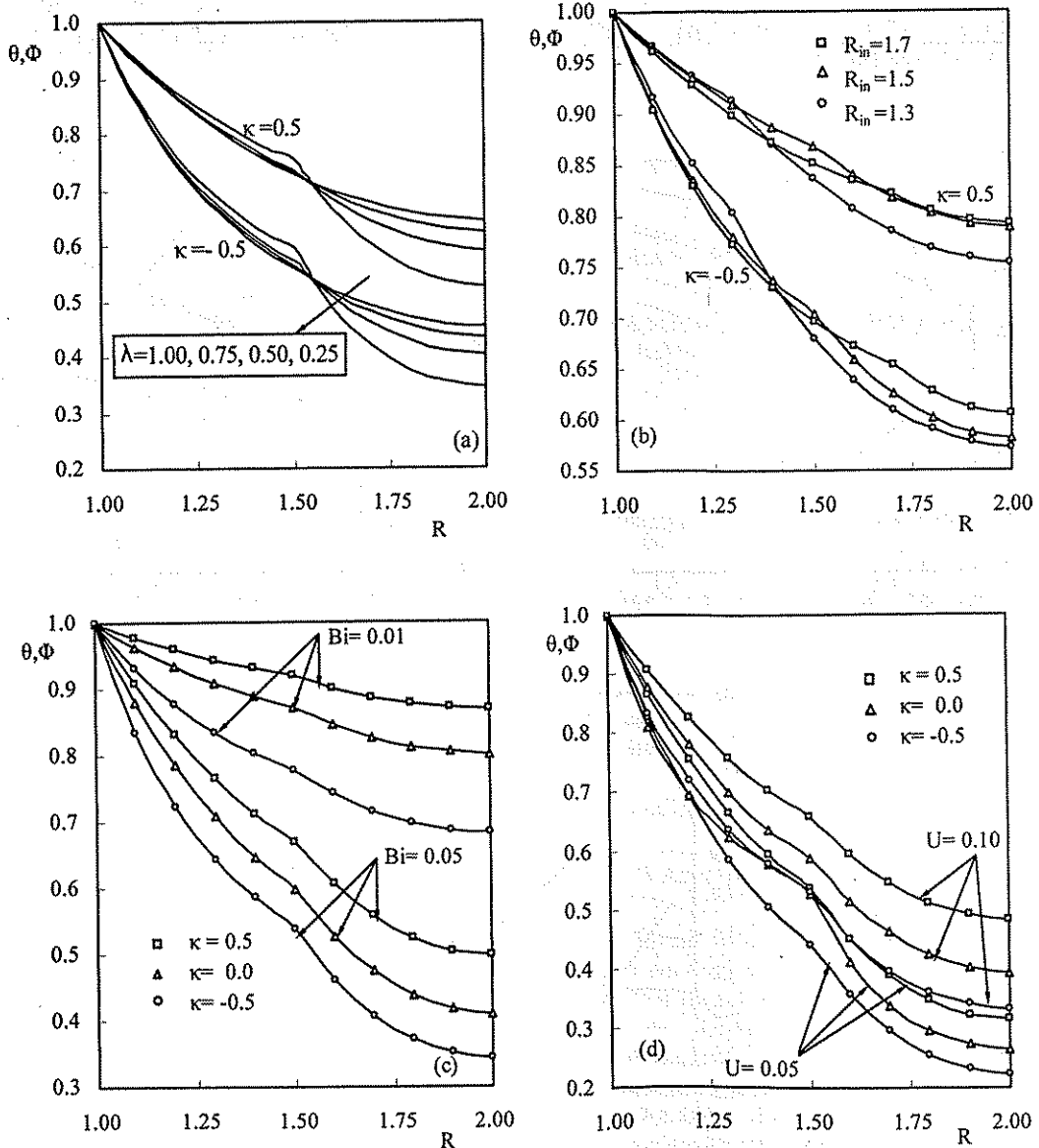


Figure 3. Temperature distribution within the annular fins with a step change in thickness ($R_o=2.0$)
(a) $R_{in}=1.5$, $\delta=0.05$, when $Bi=0.05$ (b) $\lambda=0.5$, $\delta=0.05$, when $Bi=0.03$ (c) $R_{in}=1.5$, $\lambda=0.5$, when $U=0.05$
(d) $R_{in}=1.5$, $\lambda=0.5$, when $Bi=0.1$

The dimensionless temperature distribution within the fin was calculated by taking the first five terms from the series solutions. In Figure (3a), temperature profiles of a typical annular fin with a step change in thickness for four different thickness ratio of $\lambda=0.25$, $\lambda=0.50$, $\lambda=0.75$ and $\lambda=1.00$ have been depicted, while the thermal conductivity parameter κ has two different values of $\kappa=0.5$ and $\kappa=0.3$. It will be seen that if the thickness parameter increases, the mean temperature decreases. It also can be seen that the temperature profile has an abrupt change in temperature gradient where the step change in thickness occurs. Figure (3b) shows that the value of dimensionless intermediate radius has an

important role on the temperature profile. From Figure (3c) and (3d), it will be seen that if the thermal conductivity of the fin's material increases with the temperature the mean temperature increases, i.e. $\kappa>0$. The dimensionless heat transfer rate varying with Biot number is depicted in Figure 4. It may be noted that for a given fin volume and outer radius the dimensionless heat transfer rate from an annular fin with a step change in thickness is more than that from a radial disc fin. However, the difference decreases with the decrease of Biot number.

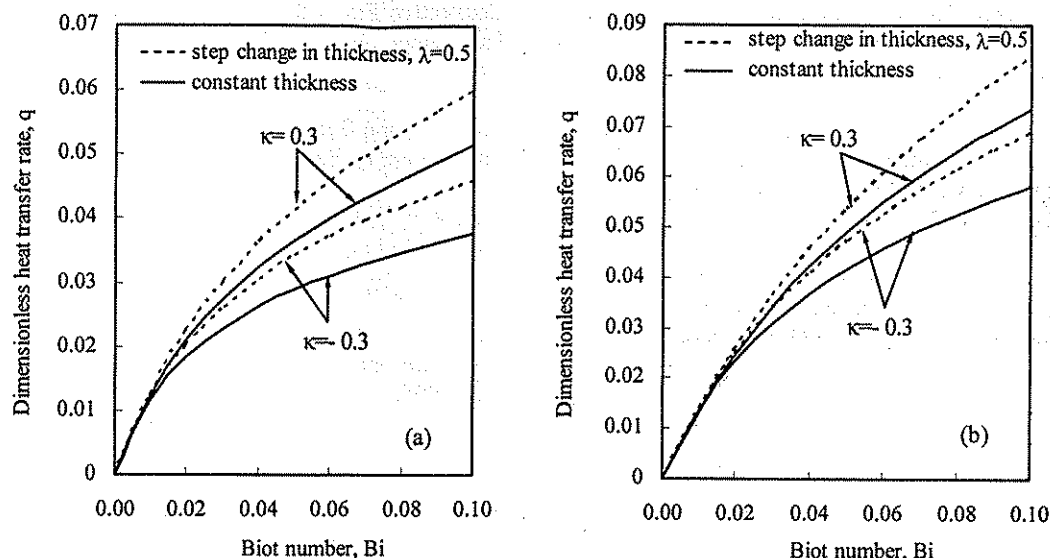


Figure 4. Dimensionless heat transfer rate as a function of Biot number for different thermal conductivity parameter ($R_o=2.0$, $R_{in}=1.5$) (a) when $U=0.05$ (b) when $U=0.10$

CONCLUSIONS

This paper has presented an essentially closed form solution for an annular fin with a step change in thickness and temperature-dependent thermal conductivity. Based on one dimensional conduction two nonlinear differential equations with nonlinear inhomogeneous boundary conditions have been obtained. The Adomian decomposition method has been used to determine the temperature distribution within the fin. The temperature distribution is continuous but experiences a change in slope where the step change in thickness occurs. Using the temperature distribution, the heat transfer rate has been obtained as a function of Biot number, geometrical fin parameters, thermal conductivity parameter describing the variation of the thermal conductivity parameter and thickness parameter which describes the variation of fin thickness. For a further thermal analysis and optimization procedure, the present analytical solution may be employed.

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